Deduction rules

$$\frac{}{\Gamma, P \vdash P, \Delta}\text{axiom} \qquad \frac{\Gamma, P \vdash \Delta \quad \Gamma \vdash P, \Delta}{\Gamma \vdash \Delta}\text{cut}$$

$$\frac{\Gamma, P, Q \vdash \Delta}{\Gamma, P \wedge Q \vdash \Delta}\ \wedge\text{-l} \qquad \frac{\Gamma \vdash P, \Delta \quad \Gamma \vdash Q, \Delta}{\Gamma \vdash P \wedge Q, \Delta}\ \wedge\text{-r}$$

$$\frac{\Gamma, \{t/x\}P \vdash \Delta}{\Gamma, \forall x\ P \vdash \Delta}\forall\text{-l} \qquad \frac{\Gamma \vdash \{c/x\}P, \Delta}{\Gamma \vdash \forall x\ P, \Delta}\forall^*\text{-r}$$

Some Rules of Sequent Calculus

Given $\mathcal{R}$ a set of rewrite rules, we add two rules to Sequent Calculus :

$$\frac{\Gamma, P \vdash_{\mathcal{R}} \Delta}{\Gamma, Q \vdash_{\mathcal{R}} \Delta}\text{rewrite-l if } P =_{\mathcal{R}} Q$$

$$\frac{\Gamma \vdash_{\mathcal{R}} P, \Delta}{\Gamma \vdash_{\mathcal{R}} Q, \Delta}\text{rewrite-r if } P =_{\mathcal{R}} Q$$

$=_{\mathcal{R}}$ is the reflexive-transitive-symmetric closure of $\rightarrow$.

Deduction rules

$$\frac{}{\Gamma, P \vdash_{\mathcal{R}} Q, \Delta}\text{axiom} \qquad \frac{\Gamma, P \vdash_{\mathcal{R}} \Delta \quad \Gamma \vdash_{\mathcal{R}} Q, \Delta}{\Gamma \vdash_{\mathcal{R}} \Delta}\text{cut}$$

$$\frac{\Gamma, P, Q \vdash_{\mathcal{R}} \Delta}{\Gamma, R \vdash \Delta}\wedge\text{-l} \qquad \frac{\Gamma \vdash_{\mathcal{R}} P, \Delta \quad \Gamma \vdash_{\mathcal{R}} Q, \Delta}{\Gamma \vdash_{\mathcal{R}} R, \Delta}\wedge\text{-r}$$

$$\frac{\Gamma, \{t/x\}P \vdash_{\mathcal{R}} \Delta}{\Gamma, Q \vdash_{\mathcal{R}} \Delta}\forall\text{-l} \qquad \frac{\Gamma \vdash_{\mathcal{R}} \{c/x\}P, \Delta}{\Gamma \vdash_{\mathcal{R}} Q, \Delta}\forall^*\text{-r}$$

Some Rules of Sequent Calculus Modulo